



INTERNATIONAL GROUP
FOR THE PSYCHOLOGY
OF MATHEMATICS EDUCATION

PROCEEDINGS

FIFTEENTH

PME CONFERENCE

ASSISI(ITALY)1991
JUNE29-JULY4

VOLUME II

PROCEEDINGS OF PME 15, ASSISI (ITALY), 1991

CONTENTS OF VOLUME II

EDWARDS L. D. - A comparison of children's learning in two Interactive Computer environments	p.1
EISENBERG T. - On building self-confidence in mathematics	p.9
ELLERTON N. - Classroom discourse and mathematics learning	p.17
ERNEST P. - Constructivism, the psychology of learning, and the nature of mathematics: some critical issues	p.25
EVANS J. T. - Cognition, affect, context in numerical activity among adults	p.33
EVEN R.; MARKOVITS Z. - Teachers' pedagogical knowledge: the case of functions	p.40
FILLOY-YAGÜE E. - Cognitive tendencies and abstraction processes in algebra learning	p.48
FURINGHETTI F.; PAOLA D. - On some obstacles in understanding mathematical texts	p.56
GOLDING.; HERSCOVICS N. - Toward a conceptual-representational analysis of the exponential function	p.64
GRAY E.; TAU. D. - Duality, ambiguity and flexibility in successful mathematical thinking	p.72
GREER B.; MC CANN M. - Children's word problems matching multiplication and division calculations	p.80
GREVSMÜHL U. - Children's verbal communication in problem solving activities	p.88
GRUGNETTI L.; MUREDDU TORRES C. - The "power" of additive structure and difficulties in ratio concept	p.96
GURTNER J. L.; VITALE B. - Why modeling? Pupils Interpretation of the activity of modelling in mathematical education	p.101
GUTIERREZ A.; JAIME A.; SHAUGHNESSY J. M.; BURGER W. F. - A comparative analysis of two ways of assessing the van Hiele levels of thinking	p.109
HALL N. - A procedural analogy theory: the role of concrete embodiments in teaching mathematics	p.117
HAREL G.; BEHR M.; POST T.; LESH R. - Variables affecting proportionality: understanding of physical principles. formation of quantitative relations, and multiplicative invariance	p.125

CHILDREN'S VERBAL COMMUNICATION IN PROBLEM SOLVING ACTIVITIES

Ulrich GREVSMÜHL,
Pädagogische Hochschule Freiburg, Germany

A model is proposed as a tool to investigate the use of children's verbal communication as an instrument of learning while working in pairs at arithmetic word problems. In an empirical investigation with a class of third year primary school children more than 2000 speech acts made by the students were classified according to the various domains of the model: cognitive, communicative, affective-emotional and social. In addition for each of the students an evaluation was made by the teachers which provided details of student's achievements and behaviour in the learning situation. The statistical analysis of the data provides further insight into the dynamics of mathematical conversations between children.

1. INTRODUCTION

In the past several models have been developed to describe and study the structures of human communication (eg Watzlawick et al 1969, Spanhel 1973, Bellack et al 1974, Bales (Piontkowski 1976), Diegritz/Rosenbusch 1977, Maier/Bauer 1978). Depending on their aims and intentions these models vary a great deal in their approach and consequently also in the dimensions which they include. Although some of the models have been applied to the special situations of teaching and learning, none of them fully account for the processes of problem solving and of communicative behaviour.

In order to investigate children's verbal communication while working in pairs at a word problem, it is therefore necessary to set up a model which comprises cognitive aspects of the problem solving process and of information processing, communicative and affective-emotional components as well as components of social interaction. In a preliminary study children's verbal communication in problem solving activities was analysed in a qualitative way where various components of communication were classified (Grevsmühl/Storbeck 1989). The purpose of the present investigation is to complete these findings by a more quantitative analysis of the dynamics of children's mathematical conversation.

In the investigation 22 students from a mixed-ability third year class of a local primary school (age 9 years) were tested in pairs outside of their regular class-setting. Five arithmetic word problems were used to promote mathematical conversations between the children. One of them is the well-known 'snail problem'¹ which has also been used in educational advisory tests (eg BBT 3-4, Beltz):

'A snail is sitting at the bottom of a well and wants to crawl out. The well is 16 meters deep. Each day the snail crawls four meters up the side of the well but during the night it slides back one meter. On which day does the snail reach the top of the well?'

At the beginning of the test the Supervisor asked one of the students to read out loud the problem. Paper and pencil was provided and the students were asked to solve the problem together. All conversations were recorded and transcribed but only 33 of them were evaluated further where there was no communication with the Supervisor and which contained a substantial amount of mathematical conversation between the students. Monologues or dialogues with predominantly private or merely organisational conversations were not included.

2. MODEL OF CHILDREN'S VERBAL COMMUNICATION IN PROBLEM SOLVING ACTIVITIES

For the investigation of children's verbal communication speech acts are evaluated on the basis of the cognitive, communicative, affective-emotional and social domains. A speech act is defined as the smallest unit of human communication carrying a content, a communicative function (relational aspect) and an intentional aspect. It may consist of only one or several words. A verbal expression may therefore consist of several speech acts. The 33 conversations contained more than 2000 speech acts which were classified according to the ten components of the model (1.1 to 4.2 of Table 1) and evaluated statistically. It was not the intention to include details of the various strategies used by the children in tackling the word problems but to investigate their mathematical conversations in a problem solving environment.

In the cognitive domain the problem solving process has been analysed by the stage model put forward by Newman and described by Watson (1980). Each of the speech

acts was not only allocated to one of the five stages but it was also recorded if the content of the speech act was correct (c), false (f) or indefinite (ind). The latter notation was also used if a correct conclusion was drawn from an incorrect thought made previously. The analysis shows that half of the speech acts occur in the transformation stage and nearly a quarter while processing skills. The majority of false or indefinite speech acts also occur in these two stages.

The component Information processing was used to record and analyse the flow of information between Speaker, partner and problem (Table 1). In more than half of the speech acts the Speaker expresses his own thoughts, about a third of which contain a new thought, nearly a third the continuation of a thought and nearly a quarter the repetition or confirmation of a thought (Table 2). In 39% of the speech acts a thought of the partner was taken up by the Speaker where in nearly half of the cases the thought was repeated by the Speaker, in 17% it was carried further and in nearly a third the partner's thought was either probed, corrected or questioned. Only in 10% of the speech acts information was taken from the word problem where in nearly half of the cases the information was provided and in a third it was repeated or confirmed.

The approaches taken by the Speakers in the conversations are for 29% of the speech acts searching-clarifying, for 41% of an arguing nature while the rest is neutral (Table 1).

For the communicative domain a linguistic analysis of the speech acts was undertaken according to the type of conversation, of language and of verbal expression. Due to the choice of conversations it was expected that more than 90% of the speech acts would be mathematical conversations and contain also a substantial amount (7.4%) of mathematical language (Table 1). A speech act was said to use mathematical language if it contains a mathematical expression or a phrase used in the teaching of mathematics. Situative language occurs in about 13% of the speech acts. These acts do not make sense on their own but only in relation to the activity and are often accompanied by nonverbal forms of communication

Verbal expressions were analysed according to a System of classification suggested by Diegritz/Rosenbusch (1977) in which each speech act is allocated to one of the classes constativa, positional or evaluativa. In constativa the speech act is task-orientated with an emphasis made on the content rather than on the relational aspect. In the class of positional the position of the Speaker becomes visible, for instance by his effort to influence the position of others. There the relational aspect is all important. The person and attitude of the Speaker are predominant and the speech act reflects more the speaker's own ideas. In the class of evaluativa the speaker pursues a subject or person orientated evaluation. Again the relational aspect plays a decisive role but in order to prevent overlapping with the class of positional an evaluativa is defined as being the reaction to a speech act made by others (eg a contradiction can only be evoked by a provocative act proceeding it). The analysis shows that 40% of all speech acts are constativa, 40% of which are Statements. Furthermore, about 30% of the speech acts are positional and about 30% are evaluativa with more than half of them being confirmations (Table 3).

For the affective-emotional and social domains the dimensions dominance, cooperation and friendliness were taken into account as suggested by Bales (Piontkowski 1976) for the analysis of interaction processes and for modelling interpersonal ratings. Whereas in nearly all of the speech acts the attitude towards the task is neutral, 6% of the speech acts show a hostile attitude towards the partner compared to a mere 1% which is friendly. In the social domain 70% of the speech acts indicate a cooperation with the partner, 8% a competition. As regards to the way in which power is executed 16% of the speech acts reflect a dominating, 8% a subservient role of the Speaker.

3. TEACHERS' EVALUATION OF STUDENTS' BEHAVIOUR

For each of the 22 students an evaluation was made by the teacher of Mathematics and independently by the teacher of German. A modified test form developed by the Institut für Bildungsplanung und Studieninformation (1975) was used which contains

achievements in Mathematics, in the writing of essays and in dictation, behaviour in the learning Situation, emotional and social behaviour.

The fairly high Pearson correlation coefficients between independence in work, ability to converse fluently and clearly and intelligible presentation of tasks indicate a common parameter within the behaviour in the learning Situation (0.71-0.95). Moderate to strong correlations were also found between these parameters, the achievements in Mathematics (0.63-0.93) and in essay (0.74-0.81) and the dominance while working in pairs in Mathematics (0.74-0.83) and in German (0.63-0.77). Furthermore, the dominance while working in pairs also correlates strongly with the achievements in Mathematics (0.83) and in essay (0.71).

4. CORRELATIONS WITH COMPONENTS OF THE MODEL

Pearson correlation coefficients were calculated between the various components of the model and also between the components and parameters of students' behaviour. As a report and discussion of all the correlations would be beyond the scope of this paper, only some of the results can be presented here.

In the Information processing (Table 1: 1.2) moderate to strong correlations were found between the total number of speech acts with the Speaker's own thought and his independence in work and dominance while working in pairs (0.6). Correlations of similar strength but negative exist between the total number of speech acts with a thought adopted from the partner and these parameters (0.6).

Positive evaluative (Table 3) correlate strongly (0.7-0.9) and positively with the number of speech acts with repetition/ confirmation of a thought adopted from the partner (Table 2) and negatively with the achievement in essay, with self-confidence and with finding contacts to others.

The constative reasoning (Table 3) correlates strongly (0.9) with correct interpretation in

Table 2: INFORMATION PROCESSING (relative frequency in %)

	speaker's own thought	thought adopted from partner	information from problem
new thought/ providing Information			
correct	19.8	-	41.2
false	14.1	-	7.7
continuing	31.0	16.9	7.3
reasoning	3.1	3.1	0.0
repeating/ confirming	22.8	48.7	33.8
probing	3.4	8.6	4.8
correcting	2.6	9.0	1.2
questioning	3.1	13.7	3.9

Table 3: TYPE OF VERBAL EXPRESSION (relative frequency in %)

Constativa			40.0
stating	40.3		
supposing	13.8		
reasoning	17.0		
answering	3.0		
reading out	8.3		
repeating	17.5		
informing	0.0		
Positional			28.7
<i>Optativa</i>		36.6	
questioning	22.3		
requesting	0.7		
reaffirming	13.6		
<i>Imperativa.</i>		20.2	
inviting	7.0		
instructing	8.0		
ordering	5.2		
<i>Constructiva</i>		14.6	
suggesting	14.6		
<i>Specific Positional</i>		28.6	
maintaining	2.8		
considering	5.9		
undecided	11.1		
persisting	8.0		
commenting	0.7		
Evaluativa			31.2
<i>Positive</i>		62.2	
confirming	52.6		
conceding	9.6		
praising	0.0		
<i>Negative</i>		37.8	
Rejecting	16.0		
Reproaching	0.6		
contradicting	6.4		
objecting	10.6		
correcting	4.2		

the problem solving process, the continuation of one's own thought and the questioning of Information gained from the problem.

The number of speech acts with an arguing approach (Table 1: 1.3) correlates strongly with the achievements in Mathematics, in essay, with independent and careful work, with self-confidence and also with finding contacts to others (0.7-0.8). On the other hand a neutral approach correlates strongly but negatively with these parameters.

5. CONCLUDING COMMENTS

There is no doubt that the results of this investigation depend on the choice of sample made (children, choice of problem and of presentations, choice of conversations, etc). Still the model itself provides important Information of the dynamics of children's verbal communication in a mathematical problem solving environment. In order to gain further insight into the dynamical aspects, it is necessary to investigate the iterative nature in which the stages of the problem solving process are passed through and also the sequence of verbal expressions within the conversations.

REFERENCES

- Bellack A / Kliebard H M / Hyman R T / Smith F L, Die Sprache im Klassenzimmer, Düsseldorf 1974
- Diegritz Th / Rosenbusch H, Kommunikation zwischen Schülern, München 1977
- Grevsmühl U / Storbeck C, Lehrersprache, Schülersprache - Aspekte sprachlicher Kommunikation im Mathematikunterricht der Grundschule, Beiträge zum Mathematikunterricht 1989, Bad Salzdetfurth, 163-166
- Institut für Bildungsplanung und Studieninformation (IBS), Merkmalsbogen zur Verhaltensbeschreibung durch den Lehrer, Stuttgart 1975
- Maier H / Bauer L, Zum Problem der Sprache im Mathematikunterricht, unpublished, Regensburg 1978
- Piontkowski U, Psychologie der Interaktion, München 1976
- Spanhel D (ed), Schülersprache und Lernprozesse, Düsseldorf 1973
- Watson I, Investigating Errors of Beginning Mathematicians, Educational Studies in Mathematics 11(3), 319-329, 1980
- Wygotski L S, Denken und Sprechen, Stuttgart 1964